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(Nota interna: n. 291)

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E.Ferlenghi: ON THE GENERALITY OF THE STABILITY CRITERION AGAINST THE TRANSVERSE WALL INSTABILITY OF RELATIVISTIC BEAMS.

Recently much attention has been given to the problem of coherent transverse instabilities induced in relativistic beams by the resistive walls of the vacuum tank of an accelerator.

In the works (1-6) the effect due to finite conductivity walls has been examined, for various types of beams, and rules have been obtained for instabilities.

The aim of the present work is to show how these rules are of quite general character and can be deduced without a detailed knowledge of the material properties of the wall. More exactly we will show that the instability rules (but not the rise times of the instabilities) are independent of the decay law of a signal impressed on the wall by a particle. This means that modifications of the material properties of the wall, obtained by operating directly on the material or by introducing structures in the vacuum tank, can modifie only the rise times. In the case of internal structure, the mechanism which may give new instability criteria should be of a quite different type (e.g. modes with a phase velocity less then c may propagate through the structure, which may be in resonance with the beam).

In order to describe the dynamical problem wa shall employ the same models and the same symbols as in the work (6). Particularly we consider that particles are moving in the longitudinal direction as $Z_i(t) = \xi_i + vt$ and that the motion in ver tical direction is described by $\eta_K e^{-i\nu t}$.

The equation of motion in the vertical direction for a particle in absence of the wall field is

$$(1) \qquad \ddot{\eta}_K + \nu_0^2 \eta_K = 0 \qquad \text{that is} \qquad (-\nu^2 + \nu_0^2)\eta_K = 0$$

where ν_0 is the betatron frequency.

The presence of the walls introduces in equations (1) a term which stands for the force due to material being not ideal. The most general mechanism, which can be assumed to describe (at every instant and in every point, independently of the type of geometry and of the material properties of the walls) the reaction field of the wall, is schematized in fig.1

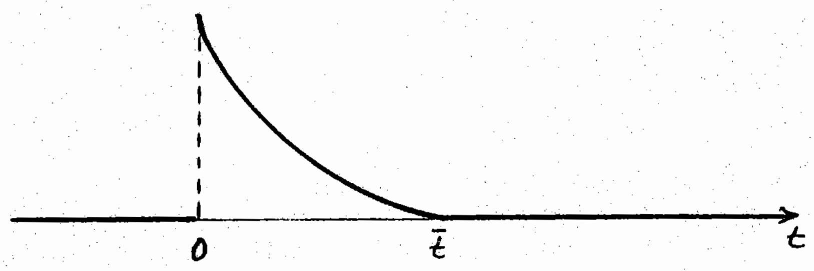

Fig.1

The signal, impressed at time t = 0 by the particle on the wall, falls off to zero in a time interval $\bar{t}$.

The force is therefore generally of the type

$$(2) \qquad P(t) = p(t)\,\theta(t)\,\theta(\bar{t}-t)\,\eta(0)$$

Obviously the function p(t) (≥ 0) is required, for any value of $\bar{t}$ (which may also tend to ∞), to fall off in a monotonic way so rapidly as to assure the convergency of all the quantities of our interest.

The initial value of the force is taken to be proportional to the vertical position of the acting particle.

From all these considerations we conclude that the equation of motion of the K-th particle in presence of other particles is

$$(3) \qquad \ddot{\eta}_K + \nu_0^2 \eta_K = \sum_i \eta_i \left(t - \frac{\xi_i - \xi_K}{v}\right) P\left(\frac{\xi_i - \xi_K}{v}\right)$$

which coincides with (4-7) of (6).

We examine here the case of a beam with only one bunch. The extension to the case of several bunches or of unbunched beam is immediate.

Taking into account the periodicity conditions in the motion of the bunch, from equation (3) one obtains the dispersion relation

$$\nu^2 = \nu_0^2 - A\,\omega_0 \sum_s P(\nu + s\omega_0) \qquad s = 0, \pm 1, \pm 2, \dots \tag{4}$$

where ω_0 is the revolution frequency and A is a positive constant in which will be included time by time all the inessential numerical coefficients(x).

By defening as in work (6),

$$P(\omega) = \text{const} \int dt\, e^{i\omega t}\, p(t)$$

we can write down again (4) as follows:

$$\nu^2 = \nu_0^2 - A\,\omega_0 \sum_s \int_0^t dt\, e^{i(\nu + s\omega_0)t}\, p(t) \tag{4'}$$

If we invert the order of the summation and the integration and remember that

$$\sum_s e^{ixs} = \text{const} \sum_s \delta(x + 2\pi s)$$

(x) - See the formula (6-2) in (6).
The extension to the cases of several bunches and of unbunched beam is obtained from the correspondent dispersion relations:

$$\nu^2 = \nu_0^2 - A\,\omega_0 \sum_s P\left[\nu + (r + sB)\omega_0\right]$$

r = 0,1,''', B-1
B is the number of bunches

and

$$\nu^2 = \nu_0^2 - A\,\omega_0\, P(\nu + s\omega_0)$$

(see resp. (6-8) and (5-2) in (6)).

(4') becomes

$$\nu^2 = \nu_0^2 - A\omega_0 \int_0^{\bar{t}} dt\; e^{i\nu t} p(t) \sum_s \delta(\omega_0 t + 2\pi s) =$$

(5) $$= \nu_0^2 - A \sum_s \int_0^{\bar{t}} dt\; e^{i\nu t} p(t)\, \delta(t + sT_0)\,.$$

In $\sum_s$ the terms with $s > o$ do not contribute, as well as, by definition, the term with s=0.

We have therefore:

(6) $$\nu^2 = \nu_0^2 - A \sum_{s=1}^{\infty} \int_0^{\bar{t}} dt\; e^{i\nu t} p(t)\, \delta(t - sT_0)$$

It is obvious that for $T_0 > \bar{t}$ we simply have $\nu^2 = \nu_0^2$

Generally $\sum_{s=1}^{\infty}$ will have as an upper bound a value $\bar{s}$, such that $\bar{s}T_0 \leq \bar{t}$.

Therefore (6) becomes:

$$\nu^2 = \nu_0^2 - A \sum_{s=1}^{\bar{s}} p(sT_0)\, e^{i\nu T_0 s}$$

and, by substituting in the right hand side $\nu \cong q\omega_0 \equiv \nu_0$,

(7) $$\nu^2 = \nu_0^2 - A \sum_{s=1}^{\bar{s}} p(sT_0)\, e^{i2\pi q s}$$

For instability, it is important to know the sign of

(8) $$Im(\nu^2) = -A \sum_{s=1}^{\bar{s}} p(sT_0) \sin(2\pi q s)$$

which is determined by q only.

Indicating the sum by S(q) we find the periodicity properties

$$S(q+1) = S(q);\; S(1-q) = S(-q) = -S(q)$$

from which

$$S(0) = S(1) = S(\tfrac{1}{2}) = 0$$

If we consider only the values $0 < q < 1$, it is also easy to see that

$$S(q) > 0 \quad \text{if} \quad 0 < q < \tfrac{1}{2}$$
$$S(q) < 0 \quad \text{if} \quad \tfrac{1}{2} < q < 1$$

from which it derives

(9)
$$\text{Im}(\nu^2) < 0 \quad \text{that is stability if} \quad 0 < q < \tfrac{1}{2}$$
$$\text{Im}(\nu^2) > 0 \quad \text{that is instability if} \quad \tfrac{1}{2} < q < 1.^{(x)}$$

The rise time of the instabilities depends obviously on the behaviour of the function p(t).

I thank F.Amman for helpfull discussions.

(x) - For the several bunches case one has

$$\text{stability} \quad \text{if} \quad 0 < \frac{q+r}{B} < \tfrac{1}{2} \quad \text{Mod } (1)$$
$$\text{instability} \quad \text{if} \quad \tfrac{1}{2} < \frac{q+r}{B} < 1 \quad \text{Mod } (1)$$

For the unbunched beam one has

$$\text{stability} \quad \text{if} \quad q + s > 0$$
$$\text{instability} \quad \text{if} \quad q + s < 0$$

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